**FIVE MATHEMATICAL GA M E S** Paul Zeitz, [zeitzp@usfca.edu](mailto:zeitzp@usfca.edu)

For all of these games, two players alternate turns. The winner is the last player who makes a legal move. See if you can find a winning strategy for one of the players. Try to prove that your strategy works. And, always, try to generalize!

1. Basic Takeaway. A set of 16 pennies is placed on a table. Two players take turns removing pennies. At each turn, a player must remove between 1 and 4 pennies (inclusive).

2. Don’t Be Greedy. Start with some pennies. The first player can take any positive number away as long as he or she doesn’t take all the pennies. After that, you must take a positive number of pennies, but you may not take more pennies than your opponent just took. For example, if you start with 20 pennies, and player A takes 5, leaving 15, then player B can take away 1, 2, 3, 4, or 5, but not 6 or more.

3. Don’t Be Doubly Greedy. This is just like Don’t Be Greedy, only now the rule is that you cannot take more than twice what your opponent just took. So if player A takes 5, player B can take any number between 1 and 10, inclusive.

4. Divide and Conquer. Start with 100 pennies. Each player can remove a divisor of the number of pennies remaining as long as the divisor is strictly less than the number of pennies remaining. For example, at the start, A could remove 1, 2, 4, 10, 20, 25, or 50 pennies, but not 100 pennies. This game ends when exactly 1 penny is left, since the only divisor of 1 is 1, which is not less than 1.

5. Puppies and Kittens. We start with a pile of 7 kittens and 10 puppies. Two players take turns; a legal move is removing any number of puppies or any number of kittens or an equal number of both puppies and kittens.

**Online Resource for More Mathematical Games and Problem Solving Activities:**

[**http://nrich.maths.org/frontpage**](http://nrich.maths.org/frontpage)

[**http://gofigurewithscipi.blogspot.com/**](http://gofigurewithscipi.blogspot.com/)

[**http://plus.maths.org/content/**](http://plus.maths.org/content/)